Generalized 3D Strong Coupled Model of Electrical Machines Closed in Loop with PI Controller

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This paper deals withfield-circuit-motion device model with PI controller closed in loop of the control system. The device model using time-stepping finite element technique is presented. Field circuit, motion and torque equations are coupled together in one global matrix system, which is nonsymmetrical and solved at each iteration step by the PBiCG algorithm. The control law given by PI controller is presented and coupled to the discrete equations of electromechanical device.The device field model is defined by magnetic vector potential, which makes distributed parameters closed-loop control system, where the device state depends not only on time but also on space configuration.

Index Terms—**PI controller, time stepping finite element technique, PBiCG, closed-loop control system.**

I. Introduction

The performance of electrical machines is defined not only by their electromagnetic or mechanical characteristics but also by control system which includes a feeding electronic circuit. Nowadays, the computation area improvements enable to analyze and develop more realistic models which can include magnetic field distribution, current flow in electric circuitry, forces, mechanical movement, as well control signals.

The researchers propose different procedures to investigate coupling the effects. They treat the problem directly coupling electric or electronic circuit equations with magnetic field equation in one global matrix system by field potentials and circuit currents which are common variables [1,2]. Also the direct coupling methods considering electronic converters, electric circuit, magnetic field and control strategy are proposed in literature [3,4].

To improve the direct coupling problem, this paper presents a smart generalized electrical machines model where electric circuit equations, magnetic field equations, motion equations and controller equations are directly coupled in one global matrix system and solved together.

II. A Strong Coupled Device Model

The device is modelled using the time-stepping finite element technique. The formulation relies on a strong coupling between magnetic field, driving circuitry and mechanical motion equations yielding a complete description of the state of the motor at every time instance during the numerical iterative process [5].

The electromagnetic field is expressed in terms of state variable the magnetic vector potential $\mathbf{A}$. The electric circuit equation for the described device is represented byinput voltages, phase currents and derivations of magnetic linkage flux as a function of magnetic vector potential $\mathbf{A}$. The electromagnetic torque or force is also a nonlinear function of magnetic vector potential $\mathbf{A}$.Using Taylor's polynomial approximation of the first degree, the torque or force can be treated as excitation in mechanical motion equation. Then magnetic vector potential $\mathbf{A}$ is a common variable and together with speed and displacement are state variables.

In the electromagnetic device model, a field-circuit, motion and torque equations can be coupled together in one global matrix system:

$$\begin{bmatrix} \mathbf{C} & \mathbf{E} & \mathbf{0} & \mathbf{0} \\ \dfrac{\mathbf{F}}{\Delta t} & \mathbf{R} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 1 & -\Delta t \\ -\Delta t \dfrac{1}{J}\mathbf{J}(\mathbf{A}^t) & \mathbf{0} & 0 & 1+\Delta t \dfrac{b}{J} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{t+\Delta t} \\ \mathbf{I}^{t+\Delta t} \\ \Theta^{t+\Delta t} \\ \omega^{t+\Delta t} \end{bmatrix} = \begin{bmatrix} \mathbf{M}^{t+\Delta t} \\ \mathbf{U}^{t+\Delta t} + \dfrac{\mathbf{F}}{\Delta t}\mathbf{A}^t \\ \Theta^t \\ \omega^t + \Delta t \dfrac{1}{J}\left[T(\mathbf{A}^t) - \mathbf{J}(\mathbf{A}^t)\mathbf{A}^t\right] \end{bmatrix}, \tag{1}$$

where$\mathbf{A}$ is the vector of the unknown magnetic vector potentials, $\mathbf{I}$ is the vector of the unknown phase currents, $\mathbf{C}$ represents the matrix related to the magnetic field, $\mathbf{R}$ is matrix related to the winding resistances, $\mathbf{E}$ is the matrix related to the winding currents, $\mathbf{F}$ is the matrix related to the linkage flux, $\mathbf{U}$ is the vector corresponding to the voltage control, $\mathbf{M}$ is the vector related to magnetization of rotating permanent magnets, Θ is a rotor angle, ω is an angular speed, $b$is adamping factor, $J$is the moment of inertia, $T(\mathbf{A}^t)$ is an electromagnetic torque and $\mathbf{J}(\mathbf{A}^t)$ is a Jacobian matrix related to electromagnetic torque.

III. Coupling a Controller

When working with electromechanical systems such electric motors, where control of the system output due to changes in the reference value or state is needed, implementation of a control algorithm may be necessary. Examples of such applications are motor speed control, position control, current control, torque or other variables.

In general, the controller can be described by state space equation and output equation

$$\begin{aligned}\frac{d\mathbf{X}}{dt} &= \mathbf{GX}+\mathbf{HE}_r\\ \mathbf{U} &= \mathbf{KX}+\mathbf{LE}_r\end{aligned}, \tag{2}$$

where $\mathbf{E}_r$ is a control error, $\mathbf{U}$ is an control variable and $\mathbf{X}$ is a controller state variable. The control error is a difference between the reference and plant variable.

$$\mathbf{E}_r = \begin{bmatrix} \mathbf{I}_{ref} - \mathbf{I} \\ \Theta_{ref} - \Theta \\ \omega_{ref} - \omega \end{bmatrix}, \tag{3}$$

The PI controller can be used to control any measurable variable, as long as this variable can be affected by manipulating some other system variables. Many control solutions have been used over the time, but the PI controller has become the industry standard due to its simplicity and good performance [6]. It is well known, that control law given by PI controller is described by

$$\mathbf{U} = \mathbf{K}_P\mathbf{E}_r + \mathbf{K}_I \int \mathbf{E}_r dt, \tag{4}$$

where the control variable is a sum of two terms: the proportional term, which is proportional to the error and integral term, which is proportional to the integral of error [7]. The description may be represented in state – space form

$$\begin{aligned}\frac{d\mathbf{X}}{dt} &= \mathbf{E}_r\\ \mathbf{U} &= \mathbf{K}_I\mathbf{X}+\mathbf{K}_P\mathbf{E}_r\end{aligned}, \tag{5}$$

which is a special case of equation (2). Considering the control error (3) the representation (5) takes the form

$$\begin{aligned}\frac{d\mathbf{X}}{dt} &= \begin{bmatrix} \mathbf{I}_{ref} - \mathbf{I} \\ \Theta_{ref} - \Theta \\ \omega_{ref} - \omega \end{bmatrix}\\ \mathbf{U} &= \mathbf{K}_I\mathbf{X} + \begin{bmatrix}\mathbf{K}_{P,I} & \mathbf{K}_{P,\Theta} & \mathbf{K}_{P,\omega}\end{bmatrix}\begin{bmatrix} \mathbf{I}_{ref} - \mathbf{I} \\ \Theta_{ref} - \Theta \\ \omega_{ref} - \omega \end{bmatrix}\end{aligned}. \tag{6}$$

Digital form of above continuous representation is achieved using backward difference approximation, which leads to following recursive equations

$$\begin{aligned}\begin{bmatrix}\tilde{\mathbf{I}} & \Delta t\mathbf{H}_1 & \Delta t\mathbf{H}_2 & \Delta t\mathbf{H}_3\end{bmatrix}\begin{bmatrix}\mathbf{X}^{t+\Delta t}\\ \mathbf{I}^{t+\Delta t}\\ \Theta^{t+\Delta t}\\ \omega^{t+\Delta t}\end{bmatrix} &= \mathbf{X}^t + \Delta t\begin{bmatrix}\mathbf{I}_{ref}\\ \Theta_{ref}\\ \omega_{ref}\end{bmatrix}\\ \mathbf{U}^{t+\Delta t} &= \mathbf{K}_I\mathbf{X}^{t+\Delta t} + \begin{bmatrix}\mathbf{K}_{P,I} & \mathbf{K}_{P,\Theta} & \mathbf{K}_{P,\omega}\end{bmatrix}\begin{bmatrix} \mathbf{I}_{ref} - \mathbf{I}^{t+\Delta t} \\ \Theta_{ref} - \Theta^{t+\Delta t} \\ \omega_{ref} - \omega^{t+\Delta t} \end{bmatrix}\end{aligned}, \tag{7}$$

where $\begin{bmatrix}\tilde{\mathbf{I}} & \Delta t\mathbf{H}_1 & \Delta t\mathbf{H}_2 & \Delta t\mathbf{H}_3\end{bmatrix} = \begin{bmatrix}\tilde{\mathbf{I}} & \Delta t\tilde{\mathbf{I}}\end{bmatrix}$. Above discrete equations coupled to the discrete equations that describe electromechanical device (8) gives the global closed - loop system of following equations

$$\begin{bmatrix} \mathbf{C} & \mathbf{E} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \frac{\mathbf{F}}{\Delta t} & \mathbf{R}+\mathbf{K}_{P,I} & \mathbf{K}_{P,\Theta} & \mathbf{K}_{P,\omega} & -\mathbf{K}_I\\ \mathbf{0} & \mathbf{0} & 1 & -\Delta t & \mathbf{0}\\ -\frac{\Delta t}{J}\mathbf{J}(\mathbf{A}^t) & \mathbf{0} & \mathbf{0} & 1+\frac{\Delta t b}{J} & \mathbf{0}\\ \mathbf{0} & \Delta t\mathbf{H}_1 & \Delta t\mathbf{H}_2 & \Delta t\mathbf{H}_3 & \tilde{\mathbf{I}} \end{bmatrix}\begin{bmatrix}\mathbf{A}^{t+\Delta t}\\ \mathbf{I}^{t+\Delta t}\\ \Theta^{t+\Delta t}\\ \omega^{t+\Delta t}\\ \mathbf{X}^{t+\Delta t}\end{bmatrix} = \begin{bmatrix} \mathbf{M}^{t+\Delta t}\\ \mathbf{K}_{P,I}\mathbf{I}_{ref} + \mathbf{K}_{P,\Theta}\Theta_{ref} + \mathbf{K}_{P,\omega}\omega_{ref} + \frac{\mathbf{F}}{\Delta t}\mathbf{A}^t\\ \Theta^t\\ \omega^t + \Delta t\frac{1}{J}\left[T(\mathbf{A}^t) - \mathbf{J}(\mathbf{A}^t)\mathbf{A}^t\right]\\ \mathbf{X}^t + \Delta t\mathbf{H}_1\mathbf{I}_{ref} + \Delta t\mathbf{H}_2\Theta_{ref} + \Delta t\mathbf{H}_3\omega_{ref} \end{bmatrix} \tag{8}$$

The presented global system of equations (8) is nonsymmetrical and solved at each iteration step by the preconditioned bi-conjugate gradient algorithm (PBiCG) dedicated for the large and sparse linear systems [8].

IV. Conclusion

In the paper, a strong coupled closed-loop model of electrical machine with PI controller is presented. Electric circuit equations, magnetic field equations, motion equations and controller equations are directly coupled in one global matrix system and solved together. In the full paper, proposed modelling technique will be verified by a numerical example of a BLDC motor control system.

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